Logical Quantifiers

1. *Subjects and Predicates:* Here is an argument:

1. Chad is a duck.
2. All ducks are rabbits.
3. Therefore, Chad is a rabbit.

We saw in unit one that this is a VALID argument (though it’s not sound). But, what would the sequent of this argument look like? Remember, in unit two we said that each letter must represent a STATEMENT—i.e., a string of words that can either be true or false. But, then, each line of the argument above is a different atomic statement. For instance, “Chad is a duck” is not a compound statement—it doesn’t have any simpler statements that it could be broken down into. (e.g., “Chad” is not a statement. It is just a name. It cannot be true or false.) So, would the sequent look like this?

**Sequent?** \( D , A \rightarrow R \)

But, how would one go about deriving “R” from “D” and “A”? The answer is that, in *Propositional Logic*, we cannot. Somehow, we need to be able to dig deeper, and be able to express the PARTS of propositions. That’s what this unit is about. In this unit, we will learn a new language: The language of *Predicate Logic*.

Take a look at statement (1): “Chad is a duck.” It has a *subject* and a *predicate*. A subject of a sentence is the thing that some property or class is being attributed to, the predicate is the property or class that is being attributed to the subject. In this unit, we’re no longer going to use capital letters to represent atomic statements. Rather, they will represent predicates; and we will use lower-case letters to represent individual subjects.

For instance, “Chad is a duck” will be expressed as “\( Dc \)”. “\( Dc \)” is an atomic statement. While this much is different, we will still use the same logical operators to connect atomic statements. Here are some examples of this notation:

- If Albert goes to the party, then Betty will go to the party. \( Pa \rightarrow Pb \)
- Charlie is either a fireman or a police officer. \( Fc \lor Pc \)
- Gary will bring chips if and only if either Ed or Frank bring dip. \( Cg \leftrightarrow (De \lor Df) \)
- Mary is happy if both Nancy and Paul are happy. \( (Hn \land Hp) \rightarrow Hm \)

*Note that each letter must represent CONSISTENTLY. For instance, in the last sentence, when the letter “H” appears three times, it must represent the SAME predication each time (i.e., “is happy” or “has the property of being happy”).*
2. The Universal Quantifier: Now take a look at statement (2) in our argument above:

2. All ducks are rabbits.

From what we’ve said so far, we might be tempted to write this as “Rd”. But, NO! This would be a mistake. Remember, lower-case letters represent INDIVIDUAL subjects. That is, they must only represent some SINGLE person, place, or thing. What “Rd” says is that some PARTICULAR duck (call her Danielle) is a rabbit—but, what statement (2) says above is that ALL ducks are rabbits. To express THAT, we’ll need a quantifier.

Once again: (2) is not really referring to a single thing. It is referring to a whole CLASS of things. It is saying that ALL of the things that are ducks are also rabbits.

If you think about that, it is really telling you that, if you look out into the world at ALL of the things, whenever you find something that is a duck, you have also found something that is a rabbit. So, what (2) really says is that, “Out of ALL of the things, if that thing is a duck, then it is also a rabbit.” Enter the universal quantifier, “∀”, which captures the word “all”. In Predicate Logic, (2) is expressed as follows:

\[(∀x)(Dx \rightarrow Rx)\]

Literally, the symbolized statement means, “For all things, if that thing is a duck, then that thing is a rabbit.” But, in this class, we will say it this way:

For all x, if x is a duck, then x is a rabbit.

Here are some more examples of this notation:

All skyscrapers are tall.
\[(∀x)(Sx \rightarrow Tx)\]

For all x, if x is a skyscraper, then x is tall.

No dogs are cats.
\[(∀x)(Dx \rightarrow \negCx)\]

For all x, if x is a dog, then x is not a cat.

All things are conceivable.
\[(∀x)Cx\]

For all x, x is conceivable.

Apples are crunchy and delicious.
\[(∀x)[Ax \rightarrow (Cx \land Dx)]\]

For all x, if x is an apple, then x is crunchy and x is delicious.

Students pass logic if and only if they take the exams.
\[(∀x)[Sx \rightarrow (Px \leftrightarrow Ex)]\]

For all x, if x is a student, then x passes logic if and only if x takes the exams.

Dogs will bite if they are frightened or harassed.
\[(∀x)[Dx \rightarrow [(Fx \lor Hx) \rightarrow Bx]]\]

For all x, if x is a dog, then, if x is either frightened or harassed, then x will bite.
3. The Existential Quantifier: So far, we’ve learned how to express the quantifiers “ALL” or “NONE”. But, then, how should we express the following argument?

1. Some of the invited guests replied.
2. All those who replied said they will attend.
3. Therefore, some of the invited guests said they will attend.

The inference from (1) and (2) to (3) seems valid. But, how do we express (1) and (3)? In other words, how do we express “SOME”? Enter the existential quantifier, “∃”, which captures the word “some”. In Predicate Logic, (1) is expressed as follows:

Some of the invited guests replied.  
\((∃x)(Gx \land Rx)\)

Note also that we will interpret “some” as “at least one”. Literally, the symbolized statement means, “There is at least one thing that is both an invited guest and someone who replied.” But, in this class, we will say it this way:

There exists an x such that x is an invited guest and x replied.

Here are some more examples of this notation:

Some classes are boring.  
\((∃x)(Cx \land Bx)\)

There exists an x such that x is a class and x is boring.

Some diseases are not curable.  
\((∃x)(Dx \land \neg Cx)\)

There exists an x such that x is a disease and x is not curable.

Unicorns do not exist.  
\(\neg(∃x)Ux\)

There does not exist an x such that x is a unicorn.

Some cats are happy if and only if they are constantly fed.  
\((∃x)[Cx \land (Hx \leftrightarrow Fx)]\)

There exists an x such that x is a cat, and x is happy if and only if x is constantly fed.

Some dogs will bite if they are frightened or harassed.  
\((∃x)[Dx \land [(Fx \lor Hx) \rightarrow Bx]]\)

There exists an x such that x is a dog, and, if x is either frightened or harassed, then x will bite.
Secret Bonus: Check out this liquor store (in Denver, Colorado):

Is this what they mean?

\((\forall x)(Bx \rightarrow Sx)\) For all x, if x is a beverage, then x is in stock at TOT\(\text{\text{	extregistered}}\)L BEVERAGE!

[Note: I later found out that the ‘\(\forall\)’ is apparently supposed to look like a beverage...]